Eur. Phys. J. C 30, 525–536 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01301-7

THE EUROPEAN PHYSICAL JOURNAL C

Clockwork SUSY: supersymmetric Ward and Slavnov–Taylor identities at work in Green's functions and scattering amplitudes

T. Ohl[1,a], J. Reuter[2,b]

[1] Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Am Hubland, 97074 Würzburg, Germany
[2] Institut für Theoretische Teilchenphysik, Universität Karlsruhe, 76128 Karlsruhe, Germany

Received: 2 June 2003 / Revised version: 1 July 2003 /
Published online: 5 September 2003 –

Abstract. We study the cancellations among Feynman diagrams that implement the Ward and Slavnov–Taylor identities corresponding to the conserved supersymmetry current in supersymmetric quantum field theories. In particular, we show that the Faddeev–Popov ghosts of gauge and supersymmetries never decouple from the physical fields, even for abelian gauge groups. The supersymmetric Slavnov–Taylor identities provide efficient consistency checks for automatized calculations and can verify the supersymmetry of Feynman rules and the numerical stability of phenomenological predictions simultaneously.

1 Introduction

Despite its excellent quantitative success, the standard model (SM) of elementary particle physics cannot describe nature up to arbitrarily high energy scales. Rather, the SM is generally considered as an effective field theory which provides an accurate description of nature up to an energy scale on the order of one TeV, but not far above. The most popular candidate for an extension of the SM is supersymmetry (SUSY), which stabilizes the extremely small ratio of the electroweak symmetry breaking scale to the Planck scale by softening ultraviolet divergences. At the same time, a SUSY scale at one TeV makes the current precision data compatible with grand unification and simultaneously provides candidates for the dark matter observed in the universe.

High energy physics experiments currently under construction for the Large Hadron Collider (LHC) and being planned at a future Electron Positron Linear Collider will discover the Higgs particle and SUSY – if they exist. However, once a Higgs boson is discovered, the determination of its quantum numbers and couplings will require precision measurements of multi-particle final states at high energies (see [1–3] for overviews).

For this purpose, precise predictions are indispensable. Obtaining such predictions typically involves the calculation of tens of thousands of contributing Feynman diagrams, both from radiative corrections and from irreducible backgrounds for many particle final states. These calculations are impossible without tools for fully automated calculations [4]. The predictions obtained with such tools must be checked for consistency: the Feynman rules and input parameters (masses, coupling constants, widths, etc.) must implement the symmetries correctly and the numerical stability of the resulting computer programs is non-trivial, since gauge and supersymmetries cause strong cancellations among the contributing diagrams. It is of course desirable to test consistency and stability also in a fully automated manner.

Since symmetries are the fundamental building blocks for the construction of specific models and simultaneously responsible for delicate cancellations among perturbative contributions, it is natural to use their consequences as consistency checks. In supersymmetric field theories we must, of course, use SUSY as one of the symmetries in addition to the ubiquitous gauge symmetries (the latter are discussed from a similar point of view in [5]).

In quantum field theories with only global symmetries, conserved currents directly lead to Ward identities (WIs) equating the divergence of a Green's function containing a current operator insertion with a sum of Green's functions of transformed fields

$$\begin{aligned}&\frac{\partial}{\partial x_\mu}\langle 0|\mathrm{T}\, j_\mu(x)\phi_1(x_1)\phi_2(x_2)\ldots|0\rangle\\&=\delta^4(x-x_1)\langle 0|\mathrm{T}\,[Q,\phi_1(x_1)]\phi_2(x_2)\ldots|0\rangle\\&\quad+\delta^4(x-x_2)\langle 0|\mathrm{T}\,\phi_1(x_1)[Q,\phi_2(x_2)]\ldots|0\rangle\\&\quad+\ldots+\langle 0|\mathrm{T}\,\partial^\mu j_\mu(x)\phi_1(x_1)\phi_2(x_2)\ldots|0\rangle,\end{aligned} \tag{1}$$

where the last term vanishes for a conserved current. In theories with local gauge symmetries, the derivation of WIs

[a] e-mail: ohl@physik.uni-wuerzburg.de
[b] e-mail: reuter@particle.uni-karlsruhe.de

breaks down in perturbation theory, because it is necessary to fix the gauge before defining the perturbative expansion. However, a global non-linear BRST symmetry survives, and the conservation of the corresponding charge generates Slavnov–Taylor identities (STIs). It is well known that the STIs provide enough constraints to make the theory well defined in all orders of perturbation theory (see e. g. [6–9]). In the special case of abelian gauge symmetries with a linear gauge-fixing condition, the STIs reduce to the WIs, as is familiar from quantum electrodynamics (QED).

For the purpose of automated consistency checks for perturbative calculations, a balance must be struck between the extreme cases of simple tests that might not be comprehensive and comprehensive tests that might become sufficiently complicated to be a source of errors themselves. With this motivation, we have studied the supersymmetric Ward identities (SWIs) and supersymmetric Slavnov–Taylor identities (SSTIs) for simple scattering amplitudes at tree level.

The rôle of the SSTIs for one-particle irreducible (1PI) vertex functions in renormalization theory has been studied comprehensively [6–10] and we cannot claim to add anything to this topic in the present paper. Indeed, most of our results could be inferred indirectly from known general theorems. On the other hand, we are not aware of a detailed demonstration of how intricately the various features of supersymmetric gauge theories interact already for very simple amplitudes. As we will show below, the SWIs and SSTIs test the Fermi statistics, the Lorentz structure of vertices and the delicate cancellations among diagrams simultaneously. These demonstrations show how the SWIs and SSTIs can be used for testing the results of automated calculations comprehensively.

In Sect. 2 we demonstrate explicitly that the SWIs are violated off the mass shell in supersymmetric gauge theories, even in the case of an abelian gauge group. We then proceed to show explicitly how the SSTIs are satisfied in supersymmetric QED (SQED) in Sect. 3 and repeat the exercise for supersymmetric Yang–Mills theory (SYM) in Sect. 4. The consistency checks presented here have been implemented in the automated optimizing matrix element generator O'Mega [11] and more examples can be found in [12].

2 Off-shell violation of supersymmetric Ward identities in gauge theories

In gauge theories with global supersymmetry, there is a Majorana spinor-valued conserved Noether current corresponding to the SUSY, which will henceforth be called the supersymmetric current or SUSY current. In the case of SQED (see Appendix B.1 for the Lagrangian and our conventions) the SUSY current reads

$$\mathcal{J}^\mu = \mathrm{i}\sqrt{2}(\phi_-\mathcal{P}_\mathrm{R} + \phi_+^\dagger\mathcal{P}_\mathrm{L})(\mathrm{i}\overleftarrow{\partial\!\!\!/} - m + eA\!\!\!/)\gamma^\mu\Psi^c$$
$$+\mathrm{i}\sqrt{2}(\phi_-^\dagger\mathcal{P}_\mathrm{L} + \phi_+\mathcal{P}_\mathrm{R})(\mathrm{i}\overleftarrow{\partial\!\!\!/} - m - eA\!\!\!/)\gamma^\mu\Psi$$
$$+\frac{1}{2}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^5 F_{\alpha\beta}\lambda - \mathrm{i}e\gamma^\mu\left(|\phi_-|^2 - |\phi_+|^2\right)\lambda\,, \quad (2)$$

where $\Psi^c \equiv \mathcal{C}\overline{\Psi}^\mathrm{T}$ is the charge conjugated fermion, i.e. the positron. This current is conserved,

$$\partial_\mu\mathcal{J}^\mu = 0, \quad (3)$$

as can be checked explicitly, using the equations of motion in the Heisenberg picture. In the quantum theory, the selectron–electron current,

$$J_\mu(p_1,p_2) = \mathrm{F.T.}\,\langle 0|\mathcal{J}_\mu(x)|\phi_-(p_1)\Psi^c(p_2)\rangle$$
$$\propto \mathcal{P}_\mathrm{R}(p\!\!\!/_1 - m)\gamma_\mu v(-p_2)\,, \quad (4)$$

provides a trivial example for an on-shell Ward identity,

$$(p_1+p_2)^\mu\mathcal{J}_\mu(p_1,p_2)$$
$$\propto \mathcal{P}_\mathrm{R}(p\!\!\!/_1 - m)\,(p\!\!\!/_1 + p\!\!\!/_2)\,v(-p_2) = 0\,, \quad (5)$$

which vanishes from the Dirac equation and the equality of electron and selectron masses. Here and in the following, we use the symbol "F.T." for the Fourier transform of Green's functions and matrix elements, suppressing δ-functions and powers of 2π from momentum conservation.

As is well known, the WI (1) is valid off-shell in non-supersymmetric QED. In order to explore the supersymmetric case, we will now discuss several examples of SWIs in SQED, writing (1) as

$$k_\mu\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\overline{\xi}\mathcal{J}^\mu(x)\mathcal{O}_1(y_1)\ldots\mathcal{O}_n(y_n)|0\rangle$$
$$= \sum_{i=1}^n \mathrm{F.T.}\langle 0|\mathrm{T}\,\mathcal{O}_1\ldots\mathcal{O}_{i-1}\delta_\xi\mathcal{O}_i(y_i)\mathcal{O}_{i+1}\ldots\mathcal{O}_n|0\rangle$$
$$\times\delta^4(x-y_i), \quad (6)$$

where k_μ is the momentum flowing into the Green's function through the current operator insertion (therefore $-k^\mu = \sum_i p_i^\mu$ is the sum over all other incoming momenta) and δ_ξ is the SUSY transformation of the fields. Note that we have multiplied the supersymmetric current in (6) by the SUSY transformation parameter ξ, turning it into a bosonic operator. In (6), we have assumed that SUSY current conservation guarantees that

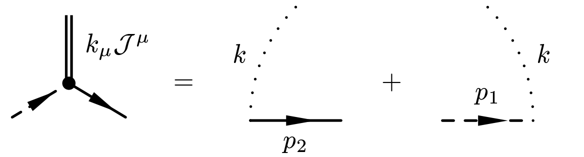
$$\Delta = \mathrm{F.T.}\langle 0|\mathrm{T}\,\partial_\mu\mathcal{J}^\mu(x)\mathcal{O}_1(y_1)\ldots\mathcal{O}_n(y_n)|0\rangle \quad (7)$$

vanishes. Unfortunately, the gauge fixing required for perturbation theory is *not* guaranteed to be compatible with SUSY current conservation. In fact, we will see soon that $\Delta \neq 0$ in Wess–Zumino gauge. Nevertheless, we will call (6) a SWI for the SUSY current, keeping in mind that any violation of (6) is equivalent to $\Delta \neq 0$.

For one selectron and one electron field, (6) reads

$$k^\mu\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\overline{\xi}\mathcal{J}_\mu(y)\phi_-^\dagger(x_1)\Psi(x_2)|0\rangle/\sqrt{2}$$
$$= \mathrm{F.T.}\langle 0|\mathrm{T}\,\Psi(x_2)\overline{\Psi}(x_1)\mathcal{P}_\mathrm{R}\xi|0\rangle\delta^4(x_1-y) \quad (8)$$
$$-\mathrm{F.T.}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)(\mathrm{i}\partial\!\!\!/ + m)\phi_-(x_2)\mathcal{P}_\mathrm{R}\xi|0\rangle\delta^4(x_2-y)\,.$$

In momentum space, the position space δ-functions in the contact terms correspond to momentum influx, which we will represent graphically by a dotted line. Using $k + p_1 + p_2 = 0$ with all momenta incoming, (8) is therefore written graphically

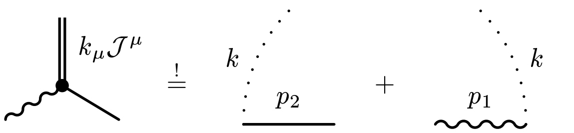


(9)

corresponding to the algebraic relation

$$\begin{aligned} &\frac{-\mathrm{i}}{p_1^2 - m^2} \frac{1}{\not p_2 + m} (\not p_1 + \not p_2) (\not p_1 + m) \mathcal{P}_\mathrm{R} \xi \\ &= \left(\frac{-\mathrm{i}}{\not p_2 + m} + \frac{-\mathrm{i}(\not p_1 + m)}{p_1^2 - m^2} \right) \mathcal{P}_\mathrm{R} \xi \,, \end{aligned} \tag{10}$$

which is indeed satisfied identically. Attempting to extend this result to the case of a photon and a photino



(11)

i.e.

$$\begin{aligned} &k^\mu \,\mathrm{F.T.} \langle 0 | \mathrm{T}\, \bar\xi \mathcal{J}_\mu(y) A_\nu(x_1) \lambda(x_2) | 0 \rangle \\ &\stackrel{!}{=} \mathrm{F.T.} \langle 0 | \mathrm{T}\, (\delta_\xi A_\nu(x_1))\, \lambda(x_2) | 0 \rangle \delta^4(x_1 - y) \\ &\quad + \mathrm{F.T.} \langle 0 | \mathrm{T}\, A_\nu(x_1)\, (\delta_\xi \lambda(x_2)) | 0 \rangle \delta^4(x_2 - y) \,, \end{aligned} \tag{12}$$

we find

$$\begin{aligned} &\frac{1}{2} k^\mu \,\mathrm{F.T.} \langle 0 | \mathrm{T}\, \lambda(x_2) \overline{\lambda}(y) \gamma^5 \gamma_\mu \left[\gamma^\alpha, \gamma^\beta \right] \partial_\alpha A_\beta(y) A_\nu(x_1) \xi | 0 \rangle \\ &\stackrel{!}{=} -\mathrm{F.T.} \langle 0 | \mathrm{T}\, \lambda(x_2) \overline{\lambda}(x_1) \gamma_\nu \gamma^5 \xi | 0 \rangle \delta^4(x_1 - y) \\ &\quad - \frac{\mathrm{i}}{2} \,\mathrm{F.T.} \langle 0 | \mathrm{T}\, A_\nu(x_1) (\partial_\alpha^{x_2} A_\beta(x_2)) \left[\gamma^\alpha, \gamma^\beta \right] \gamma^5 \xi | 0 \rangle \\ &\quad \times \delta^4(x_2 - y) \end{aligned} \tag{13}$$

and

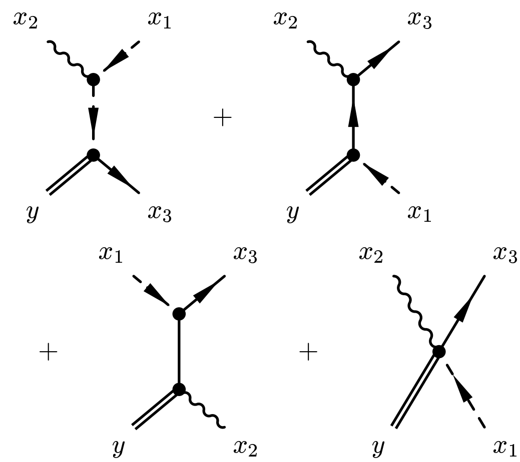
$$\begin{aligned} &\frac{1}{2} (-1) (p_1^\mu + p_2^\mu) \frac{-1}{\not p_2} \gamma^5 \gamma_\mu \left[\gamma^\alpha, \gamma^\beta \right] (-\mathrm{i} p_{1,\alpha}) \frac{-\mathrm{i} \eta_{\beta\nu}}{p_1^2} \xi \\ &\stackrel{!}{=} \frac{1}{\not p_2} \gamma_\nu \gamma^5 \xi - \frac{1}{2} \frac{1}{p_1^2} [-\not p_1, \gamma_\nu] \gamma^5 \xi \,. \end{aligned} \tag{14}$$

After some algebra, we can rewrite the left-hand side of (14)

$$\frac{1}{2} \frac{1}{p_1^2} [\not p_1, \gamma_\nu] \gamma^5 \xi + \frac{1}{\not p_2} \gamma_\nu \gamma^5 \xi - \frac{1}{\not p_2} \frac{\not p_1}{p_1^2} p_{1,\nu} \gamma^5 \xi \tag{15}$$

and the SWI (6) is not satisfied off-shell (see also [13]). We did not expect this violation of a SWI, i. e. $\Delta \neq 0$, for a global symmetry in an abelian gauge theory. We notice that this violation is proportional to the momentum of the gauge boson. Therefore it vanishes for physical matrix elements and the SWI is valid on-shell.

Before discussing the physics of this violation of the SWI, we accumulate more evidence. At tree level, there are four Feynman diagrams contributing to the matrix element of the supersymmetric current for a photon, a selectron and an electron:



(16)

Introducing the amputated Green's function

$$\begin{aligned} &\mathrm{F.T.} \langle 0 | \mathrm{T}\, \overline{\mathcal{J}_\mu}(y) \xi \phi_-^\dagger(x_1) A_\nu(x_2) \Psi(x_3) | 0 \rangle \\ &= \frac{\mathrm{i}}{p_1^2 - m^2} \frac{-\mathrm{i}}{p_2^2} \frac{-\mathrm{i}}{\not p_3 + m} \\ &\quad \times \mathrm{F.T.} \langle 0 | \mathrm{T}\, \overline{\mathcal{J}_\mu}(y) \phi_-^\dagger(x_1) A_\nu(x_2) \Psi(x_3) | 0 \rangle_{\mathrm{amp.}} \xi \end{aligned} \tag{17}$$

we find

$$\begin{aligned} &\mathrm{F.T.} \langle 0 | \mathrm{T}\, \overline{\mathcal{J}_\mu}(y) \phi_-^\dagger(x_1) A_\nu(x_2) \Psi(x_3) | 0 \rangle_{\mathrm{amp.}} \xi \\ &= -\frac{\sqrt{2} \mathrm{i} e (2p_1 + p_2)_\nu}{(p_1 + p_2)^2 - m^2} \gamma_\mu \left(\not p_1 + \not p_2 + m \right) \xi_\mathrm{R} \\ &\quad + \sqrt{2} \mathrm{i} e \gamma_\nu \frac{1}{\not p_2 + \not p_3 + m} \gamma_\mu (\not p_1 + m) \xi_\mathrm{R} \\ &\quad + \frac{\mathrm{i} e}{\sqrt{2}} \mathcal{P}_\mathrm{R} \frac{1}{\not p_1 + \not p_3} \gamma^5 \gamma_\mu [\not p_2, \gamma_\nu] + \sqrt{2} \mathrm{i} e \gamma_\mu \gamma_\nu \xi_\mathrm{R} \,. \end{aligned} \tag{18}$$

On the other hand, there are four non-vanishing contributions from the SUSY transformations of these fields

$$\begin{aligned} &\mathrm{F.T.} \sqrt{2} \langle 0 | \mathrm{T}\, \overline{\Psi}(x_1) \xi_\mathrm{R} A_\nu(x_2) \Psi(x_3) | 0 \rangle \\ &= \sqrt{2} e \frac{1}{p_2^2} \frac{1}{\not p_3 + m} \gamma_\nu \frac{1}{\not p_1 + \not k - m} \xi_\mathrm{R} \,, \end{aligned} \tag{19a}$$

$$\begin{aligned} &\mathrm{F.T.} \langle 0 | \mathrm{T}\, \phi_-^\dagger(x_1) (-\bar\xi \gamma_\nu \gamma^5 \lambda(x_2)) \Psi(x_3) | 0 \rangle \\ &= -\sqrt{2} e \frac{1}{p_1^2 - m^2} \frac{1}{\not p_3 + m} \frac{1}{\not p_2 + \not k} \gamma_\nu \xi_\mathrm{R} \,, \end{aligned} \tag{19b}$$

$$
\begin{aligned}
&- \mathrm{F.T.}\sqrt{2}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)A_\nu(x_2)(\mathrm{i}\partial\!\!\!/ + m)\phi_-(x_3)\xi_\mathrm{R})|0\rangle \\
&= -\sqrt{2}e\frac{1}{p_1^2-m^2}\frac{1}{p_2^2}(p_{3,\nu}-p_{1,\nu}+k_\nu)\frac{1}{((p_3+k)^2-m^2)} \\
&\quad\times(-p\!\!\!/_3 - k\!\!\!/ + m)\xi_\mathrm{R}\,, \qquad (19\mathrm{c})
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{F.T.}\sqrt{2}e\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)A_\nu(x_2)(-\gamma^\mu(A_\mu\phi_-)(x_3))\xi_\mathrm{R})|0\rangle \\
&= -\sqrt{2}e\frac{1}{p_1^2-m^2}\frac{\eta_{\mu\nu}}{p_2^2}\gamma^\mu\xi_\mathrm{R}\,, \qquad (19\mathrm{d})
\end{aligned}
$$

where (19d) includes a composite operator insertion from the non-linear SUSY transformation. Contracting the current matrix element with $k_\mu = -(p_1+p_2+p_3)_\mu$, we find after some algebra

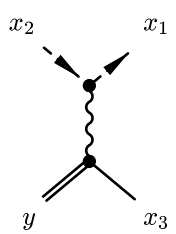
$$
\begin{aligned}
&\frac{-\mathrm{i}}{p\!\!\!/_3+m}\frac{1}{(p_1^2-m^2)p_2^2} \\
&\quad\times k^\mu\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\overline{\mathcal{J}_\mu}(y)\phi_-^\dagger(x_1)A_\nu(x_2)\Psi(x_3)|0\rangle_{\mathrm{amp.}}\xi \\
&\quad= \text{contact terms (19)} \\
&\qquad+\frac{\sqrt{2}e}{(p_1^2-m^2)p_2^2(p\!\!\!/_3+m)} \\
&\qquad\quad\times\Big\{-(p\!\!\!/_1+p\!\!\!/_2-m)\gamma_\nu+(2p_1+p_2)_\nu \\
&\qquad\quad-\gamma_\nu(p\!\!\!/_1+m)+\frac{1}{2}[p\!\!\!/_2,\gamma_\nu]-\frac{p_{2,\nu}}{p\!\!\!/_1+p\!\!\!/_3}\Big\}\xi_\mathrm{R}\,, \qquad (20)
\end{aligned}
$$

and the term violating the SWI off-shell is again proportional to the momentum of the gauge boson and vanishes on-shell.

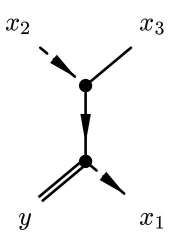
Before jumping to the conclusion that the off-shell violation of the SWI is connected to the presence of external gauge bosons, we should consider yet another example that does not contain any gauge bosons:

$$
\begin{aligned}
&k^\mu\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\bar\xi\mathcal{J}_\mu(y)\phi_-^\dagger(x_1)\phi_-(x_2)\lambda(x_3)|0\rangle \\
&\overset{?}{=} \mathrm{F.T.}\sqrt{2}\langle 0|\mathrm{T}\,(\overline{\Psi_\mathrm{L}}(x_1)\xi_\mathrm{R})\phi_-(x_2)\lambda(x_3)|0\rangle\delta^4(x_1-y) \\
&\quad+\mathrm{F.T.}\sqrt{2}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)(\overline{\xi_\mathrm{R}}\Psi_\mathrm{L}(x_2))\lambda(x_3)|0\rangle\delta^4(x_2-y) \\
&\quad-\frac{\mathrm{i}}{2}\mathrm{F.T.}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)\phi_-(x_2)\partial_\alpha A_\beta(x_3)\left[\gamma^\alpha,\gamma^\beta\right]\gamma^5\xi|0\rangle \\
&\qquad\times\delta^4(x_3-y) \\
&\quad-e\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)\phi_-(x_2)(\phi_-^\dagger\phi_-)(x_3)\xi|0\rangle \\
&\qquad\times\delta^4(x_3-y)\,. \qquad (21)
\end{aligned}
$$

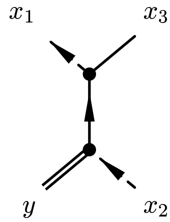
There are again four diagrams contributing at tree level to the Green's function with current insertion:



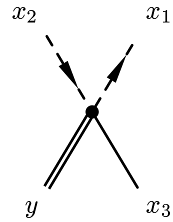
+



+



+



(22)

and we find for the left-hand side of (21)

$$
\begin{aligned}
&k^\mu\frac{e}{(p_1^2-m^2)(p_2^2-m^2)p\!\!\!/_3} \\
&\quad\times\Big\{-\frac{1}{(p_1+p_2)^2}\gamma_\mu[p\!\!\!/_1,p\!\!\!/_2]\gamma^5\xi \\
&\quad+\mathcal{P}_\mathrm{L}\frac{2}{p\!\!\!/_1+p\!\!\!/_3+m}\gamma_\mu(p\!\!\!/_2+m)\xi_\mathrm{R} \\
&\quad+\mathcal{P}_\mathrm{R}\frac{2}{p\!\!\!/_2+p\!\!\!/_3+m}\gamma_\mu(p\!\!\!/_1+m)\xi_\mathrm{L}-\gamma_\mu\xi\Big\} \\
&= -\frac{2e}{(p_1^2-m^2)p\!\!\!/_3}\mathcal{P}_\mathrm{L}\frac{1}{p\!\!\!/_1+p\!\!\!/_3+m}\xi_\mathrm{R} \\
&\quad-\frac{2e}{(p_2^2-m^2)p\!\!\!/_3}\mathcal{P}_\mathrm{R}\frac{1}{p\!\!\!/_2+p\!\!\!/_3+m}\xi_\mathrm{L} \\
&\quad+\frac{e}{(p_1^2-m^2)(p_2^2-m^2)}\xi \qquad (23) \\
&\quad+\frac{e}{(p_1^2-m^2)(p_2^2-m^2)(p_1+p_2)^2}[p\!\!\!/_1,p\!\!\!/_2]\gamma^5\xi \\
&\quad+\frac{e}{(p_1^2-m^2)(p_2^2-m^2)p\!\!\!/_3}\frac{1}{p\!\!\!/_1+p\!\!\!/_2}(p_1^2-p_2^2)\gamma^5\xi\,.
\end{aligned}
$$

In the first four terms on the right-hand side we recognize the contact terms, but the last one violates the SWI (21) off-shell

$$
\begin{aligned}
&k^\mu\,\mathrm{F.T.}\langle 0|\mathrm{T}\,\mathcal{J}_\mu\phi_-^\dagger(x_1)\phi_-(x_2)\lambda(x_3)|0\rangle \\
&-\text{contact terms}\propto(p_1^2-p_2^2) \qquad (24)
\end{aligned}
$$

and vanishes on-shell by the equality of the selectron and anti-selectron masses.

Summarizing our observations for these SQED examples, we find that the SWIs are indeed satisfied on-shell, as expected. However, we also find that even for an abelian gauge theory, the SWIs must not be continued off the mass shell. Once we are aware of this problem, we could avoid it in the practical application of testing matrix elements (and automated matrix element generators). We can either use Green's functions with more legs on-shell instead of Green's functions with fewer legs off-shell or go to the SSTIs discussed below.

However, there remains the theoretical question: why are the SWIs violated off-shell even for abelian gauge theories, contrary to the *naive* extrapolation from QED that $\Delta = 0$ in (7)? As has been shown in [9, 10, 13], SUSY is *not* a symmetry of the S-matrix for perturbative SUSY gauge theories. The gauge-fixing procedure required for the quantization of gauge theories is not compatible with SUSY and breaks the invariance of the action under SUSY. Therefore, the SWIs are not valid in the whole indefinite metric "Hilbert" space used for the covariant quantization of gauge theories, but only in its physical subspace. By the same token, the SUSY charge does not commute with the S-operator in supersymmetric gauge theories. However, the difference of the action of the SUSY charge operator on the space of asymptotic "in" and asymptotic "out" states can be written as the combined gauge and SUSY BRST transformation of the derivative of the effective action with respect to the ghost of SUSY [9]

$$Q_{\text{out}} - Q_{\text{in}} = \mathrm{i}\left[Q_{\text{BRST}}, \frac{\partial \varGamma_{\text{eff}}}{\partial \bar{\epsilon}}\right] . \tag{25}$$

In the language of [6], this can be rewritten as an identity for the commutator of the SUSY charge with the S-operator

$$[Q_{\text{in}}, S] = -\mathrm{i}\left[Q_{\text{BRST}}, \frac{\partial \varGamma_{\text{eff}}}{\partial \bar{\epsilon}} \circ S\right] , \tag{26}$$

where the symbol "$\circ$" denotes operator insertion. The right-hand side vanishes between physical states, which span the cohomology of the BRST charge. Therefore, the SUSY charge is indeed a conserved symmetry operator on the physical Hilbert space, but not on the larger indefinite metric space.

3 Supersymmetric Slavnov–Taylor identities in SQED

The Lagrangian of SQED, given in Appendix B.1 is invariant under the BRST transformation s:

$$s\phi_-(x) = \mathrm{i}ec(x)\phi_-(x) - \sqrt{2}\,(\bar{\epsilon}\mathcal{P}_{\mathrm{L}}\varPsi(x)) - \mathrm{i}\omega^\nu\partial_\nu\phi_-(x)\,, \tag{27a}$$

$$s\phi_-^\dagger(x) = -\mathrm{i}ec(x)\phi_-^\dagger(x) + \sqrt{2}\,(\overline{\varPsi}(x)\mathcal{P}_{\mathrm{R}}\epsilon) - \mathrm{i}\omega^\nu\partial_\nu\phi_-^\dagger(x)\,, \tag{27b}$$

$$s\phi_+(x) = -\mathrm{i}ec(x)\phi_+(x) + \sqrt{2}\,(\overline{\varPsi}(x)\mathcal{P}_{\mathrm{L}}\epsilon) - \mathrm{i}\omega^\nu\partial_\nu\phi_+(x)\,, \tag{27c}$$

$$s\phi_+^\dagger(x) = +\mathrm{i}ec(x)\phi_+^\dagger(x) - \sqrt{2}\,(\bar{\epsilon}\mathcal{P}_{\mathrm{R}}\varPsi(x)) - \mathrm{i}\omega^\nu\partial_\nu\phi_+^\dagger(x)\,, \tag{27d}$$

$$\begin{aligned} s\varPsi(x) &= \mathrm{i}ec(x)\varPsi(x) \\ &\quad + \sqrt{2}\Big[(\mathrm{i}\partial\!\!\!/ + m)\phi_-(x)\mathcal{P}_{\mathrm{R}} - (\mathrm{i}\partial\!\!\!/ + m)\phi_+^\dagger(x)\mathcal{P}_{\mathrm{L}} \\ &\quad + eA\!\!\!/(x)\phi_-(x)\mathcal{P}_{\mathrm{R}} - eA\!\!\!/(x)\phi_+^\dagger(x)\mathcal{P}_{\mathrm{L}}\Big]\epsilon \\ &\quad - \mathrm{i}\omega^\nu\partial_\nu\varPsi(x)\,, \end{aligned} \tag{27e}$$

$$\begin{aligned} s\overline{\varPsi}(x) &= -\mathrm{i}ec(x)\overline{\varPsi}(x) \\ &\quad + \sqrt{2}\bar{\epsilon}\Big[\mathcal{P}_{\mathrm{L}}(\mathrm{i}\partial\!\!\!/ - m)\phi_-^\dagger(x) - \mathcal{P}_{\mathrm{R}}(\mathrm{i}\partial\!\!\!/ - m)\phi_+(x) \\ &\quad - e\phi_-^\dagger(x)\mathcal{P}_{\mathrm{L}}A\!\!\!/(x) + e\phi_+(x)\mathcal{P}_{\mathrm{R}}A\!\!\!/(x)\Big] \\ &\quad - \mathrm{i}\omega^\nu\partial_\nu\overline{\varPsi}(x)\,, \end{aligned} \tag{27f}$$

$$sA_\mu(x) = \partial_\mu c(x) - \bar{\epsilon}\gamma_\mu\lambda(x) - \mathrm{i}\omega^\nu\partial_\nu A_\mu(x)\,, \tag{27g}$$

$$s\lambda(x) = \frac{\mathrm{i}}{2}F_{\alpha\beta}(x)\gamma^\alpha\gamma^\beta\epsilon + e\,|\phi_-(x)|^2\,\gamma^5\epsilon - e\,|\phi_+(x)|^2\,\gamma^5\epsilon - \mathrm{i}\omega^\nu\partial_\nu\lambda(x)\,, \tag{27h}$$

$$s\overline{\lambda}(x) = -\frac{\mathrm{i}}{2}\bar{\epsilon}\gamma^\alpha\gamma^\beta F_{\alpha\beta}(x) + e\bar{\epsilon}\gamma^5\,|\phi_-(x)|^2 - e\bar{\epsilon}\gamma^5\,|\phi_+(x)|^2 - \mathrm{i}\omega^\nu\partial_\nu\overline{\lambda}(x)\,, \tag{27i}$$

$$sc(x) = \mathrm{i}(\bar{\epsilon}\gamma^\mu\epsilon)A_\mu(x) - \mathrm{i}\omega^\nu\partial_\nu c(x)\,, \tag{27j}$$

$$s\bar{c}(x) = \mathrm{i}B(x) - \mathrm{i}\omega^\nu\partial_\nu\bar{c}(x)\,, \tag{27k}$$

$$sB(x) = (\bar{\epsilon}\gamma^\mu\epsilon)\partial_\mu\bar{c}(x) - \mathrm{i}\omega^\nu\partial_\nu B(x)\,, \tag{27l}$$

$$s\epsilon = 0\,, \tag{27m}$$

$$s\omega^\mu = (\bar{\epsilon}\gamma^\mu\epsilon) \tag{27n}$$

The identities for adjoint fields follow from the relations

$$sB^\dagger = (sB)^\dagger, \qquad sF^\dagger = -(sF)^\dagger. \tag{28}$$

for bosonic fields B and fermionic fields F.

In addition to the familiar Faddeev–Popov ghosts for the abelian gauge symmetry $c(x)$, $\bar{c}(x)$, there are ghosts for SUSY ϵ and for translations ω^μ. Since we are only considering global SUSY, the ghosts ϵ and ω^μ are constants, which will later allow a simple power series expansion of SSTIs with respect to these ghosts. Our conventions for the ghosts are spelled out in Appendix C, but we should stress here that ϵ is bosonic, because it is a ghost for a fermionic symmetry. The transformations of the ghosts are chosen to guarantee the closure of the algebra [9, 14] and can be understood from an examination of the super-Poincaré algebra. The first part of each transformation in (27) – if present – stems from the gauge transformation, the second from the SUSY transformation and the last from the translation.

As required for a BRST transformation, the transformation (27) is manifestly nilpotent

$$\begin{aligned} s^2\phi_- = s^2\phi_+ = s^2A_\mu = s^2c = s^2\bar{c} \\ = s^2B = s^2\epsilon = s^2\omega_\mu = 0\,, \end{aligned} \tag{29a}$$

except for the transformation of the fermion fields, where the square of the BRST operator is proportional to their equations of motion:

$$s^2\varPsi = -\frac{1}{2}(\bar{\epsilon}\gamma^\mu\epsilon)\gamma_\mu\frac{\delta\varGamma}{\delta\overline{\varPsi}}\,, \tag{29b}$$

$$s^2\lambda = -\frac{1}{4}(\bar{\epsilon}\gamma^\mu\epsilon)\gamma_\mu\frac{\delta\varGamma}{\delta\overline{\lambda}}\,. \tag{29c}$$

The derivation of the latter identities requires multiple use of the Fierz identities.

The gauge-fixing and ghost terms have the form

$$S_{\mathrm{GF+FP}} = -\mathrm{i}\int \mathrm{d}^4x\, s(\bar{c}F) \tag{30}$$

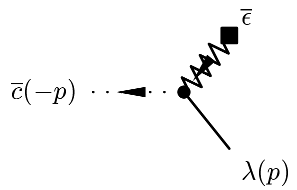
with a gauge-fixing function F. For definiteness, we will choose a class of linear and covariant gauge-fixing functions

$$F = \partial^\mu A_\mu + \frac{\xi}{2}B\,, \tag{31}$$

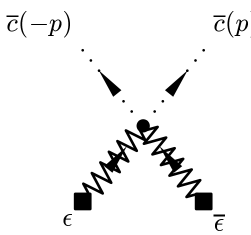
with a free gauge parameter ξ and Nakanishi–Lautrup auxiliary field B. The BRST transformation yields

$$S_{\mathrm{GF+FP}} = \int \mathrm{d}^4x \left\{ B\partial_\mu A^\mu + \frac{\xi}{2}B^2 + \mathrm{i}\bar{c}\Box c - \mathrm{i}\bar{c}(\bar{\epsilon}\partial\!\!\!/\lambda) + \mathrm{i}\frac{\xi}{2}\bar{c}(\bar{\epsilon}\gamma^\mu\epsilon)\partial_\mu\bar{c} \right\}. \tag{32}$$

The last two terms, which are absent in non-supersymmetric QED, couple photino, Faddeev–Popov ghosts and SUSY ghosts:



$$= -\mathrm{i}p\!\!\!/ \tag{33a}$$



$$= \xi p\!\!\!/ \tag{33b}$$

As we shall see in Sect. 3.1, these couplings are crucial for the SSTIs. Intuitively, SUSY and gauge symmetry don't commute in the de Wit–Freedman description [15] and even abelian gauge models become necessarily non-abelian.

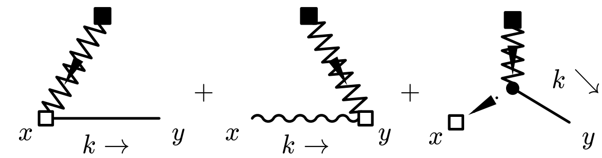
Note that only the gauge ghosts are propagating fields, while all other ghosts are simply constant operator insertions. Black boxes in the Feynman diagrams indicate the ends of ghost lines.

3.1 Examples for Slavnov–Taylor identities in SQED

In this section we will explicitly demonstrate exemplary SSTIs for SQED[1]. Starting with the case of photon and photino [8, 13, 16] which heralded the problems with the SWI in (12)

$$\langle 0|\mathrm{T}\,\{Q_{\mathrm{BRST}}, A_\nu(x)\lambda(y)\}|0\rangle = 0\,, \tag{34}$$

there are three contributing diagrams



(35)

The first diagram evaluates to

$$-\langle 0|\mathrm{T}\,(\bar{\epsilon}\gamma_\nu\lambda(x))\lambda(y)|0\rangle = +\langle 0|\mathrm{T}\,\lambda(y)(\bar{\lambda}(x)\gamma_\nu\epsilon)|0\rangle \xrightarrow{\text{F.T.}} \frac{\mathrm{i}}{k\!\!\!/}\gamma_\nu\epsilon\,, \tag{36}$$

where the fact that the SUSY ghost ϵ is commuting enters via $\bar{\epsilon}\gamma_\nu\lambda = +\bar{\lambda}\gamma_\nu\epsilon$ and $(\bar{\lambda}\gamma_\nu\epsilon)\lambda = -\lambda(\bar{\lambda}\gamma_\nu\epsilon)$. The second diagram evaluates to

$$\frac{\mathrm{i}}{2}\langle 0|\mathrm{T}\,A_\nu(x)F_{\alpha\beta}(y)\gamma^\alpha\gamma^\beta\epsilon|0\rangle \xrightarrow{\text{F.T.}} \frac{\mathrm{i}}{2}\frac{-\mathrm{i}\eta_{\nu\beta}}{k^2}(-\mathrm{i}k_\alpha)\left[\gamma^\alpha,\gamma^\beta\right]\epsilon = -\frac{\mathrm{i}}{2}\frac{1}{k^2}[k\!\!\!/,\gamma_\nu]\epsilon\,. \tag{37}$$

Finally, the third diagram contains one interaction operator from (32), which carries no coupling constant:

$$\langle 0|\mathrm{T}\,\partial_\nu^x c(x)\lambda(y)|0\rangle = -\int \mathrm{d}^4z\,\langle 0|\mathrm{T}\,\partial_\nu^x c(x)\bar{c}(z)\lambda(y)(\bar{\lambda}(z)\overleftarrow{\partial\!\!\!/}_z\,\epsilon)|0\rangle \xrightarrow{\text{F.T.}} -\frac{-1}{k^2}(\mathrm{i}k_\nu)\frac{\mathrm{i}}{k\!\!\!/}(\mathrm{i}k\!\!\!/)\epsilon = -\frac{\mathrm{i}k_\nu}{k^2}\,, \tag{38}$$

and the three terms add to zero

$$\frac{\mathrm{i}}{k\!\!\!/}\gamma_\nu\epsilon - \frac{\mathrm{i}}{2}\frac{1}{k^2}[k\!\!\!/,\gamma_\nu]\epsilon - \frac{\mathrm{i}k_\nu}{k^2} = 0\,, \tag{39}$$

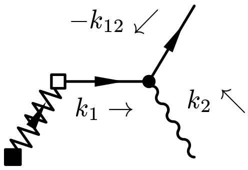
without application of the equations of motion. In contrast to the SWI, the SSTI is therefore fulfilled off-shell. Obviously, the term coupling the two ghosts to the photino is crucial here.

Encouraged by this observation, we can turn to more complicated examples. For electron, selectron and photon (cf. (20)) we should find

$$\begin{aligned} 0 &= \langle 0|\mathrm{T}\left\{Q_{\mathrm{BRST}}, \phi_-^\dagger(x_1)A_\nu(x_2)\Psi(x_3)\right\}|0\rangle \\ &= -\mathrm{i}e\langle 0|\mathrm{T}\,c(x_1)\phi_-^\dagger(x_1)A_\nu(x_2)\Psi(x_3)|0\rangle \\ &\quad +\sqrt{2}\langle 0|\mathrm{T}\,(\bar{\Psi}(x_1)\mathcal{P}_{\mathrm{R}}\epsilon)A_\nu(x_2)\Psi(x_3)|0\rangle \\ &\quad +\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)(\partial_\nu c(x_2))\Psi(x_3)|0\rangle \\ &\quad -\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)(\bar{\lambda}(x_2)\gamma_\nu\epsilon)\Psi(x_3)|0\rangle \\ &\quad +\mathrm{i}e\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)A_\nu(x_2)c(x_3)\Psi(x_3)|0\rangle \\ &\quad +\langle 0|\mathrm{T}\Big[\phi_-^\dagger(x_1)A_\nu(x_2) \\ &\qquad \times\sqrt{2}\big((\mathrm{i}\partial\!\!\!/ + m)\phi_-(x_3)\mathcal{P}_{\mathrm{R}} + (\mathrm{i}\partial\!\!\!/ - m)\phi_+^\dagger(x_3)\mathcal{P}_{\mathrm{L}} \\ &\qquad +eA\!\!\!/(x_3)(\phi_-(x_3)\mathcal{P}_{\mathrm{R}} + \phi_+^\dagger(x_3)\mathcal{P}_{\mathrm{L}})\big)\Big]\epsilon|0\rangle\,. \end{aligned} \tag{40}$$

[1] For brevity, all formulae are given in Feynman gauge $\xi = 1$, but all results have been verified for arbitrary $\xi \neq 1$ as well.

The first and the penultimate Green's function vanish, and the second Green's function yields graphically

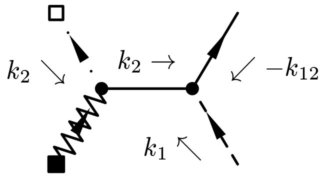


(41)

and analytically

$$\begin{aligned}&-\sqrt{2}\frac{(-\mathrm{i}\eta_{\nu\beta})}{k_2^2}\frac{\mathrm{i}}{\not{k}_{12}-m}\left(\mathrm{i}e\gamma^{\beta}\right)\frac{\mathrm{i}}{\not{k}_1-m}\mathcal{P}_{\mathrm{R}}\epsilon\\ &=\frac{\sqrt{2}e}{k_2^2}\frac{1}{\not{k}_{12}-m}\gamma_\nu\frac{1}{\not{k}_1-m}\mathcal{P}_{\mathrm{R}}\epsilon \qquad (42)\end{aligned}$$

where we have introduced the shorthand $k_{12}=k_1+k_2$. In the third Green's function, the ghost interaction contributes again

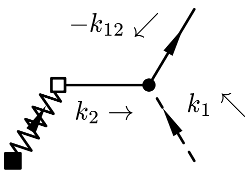


(43)

and yields

$$\begin{aligned}&-(\mathrm{i}k_{2,\nu})\frac{\mathrm{i}}{k_1^2-m^2}\frac{\mathrm{i}}{\not{k}_{12}-m}\left(-\mathrm{i}e\sqrt{2}\mathcal{P}_{\mathrm{R}}\right)\frac{\mathrm{i}}{\not{k}_2}\mathrm{i}\not{k}_2\frac{-1}{k_2^2}\epsilon\\ &=+\sqrt{2}\frac{ek_{2,\nu}}{(k_1^2-m^2)\,k_2^2}\frac{1}{\not{k}_{12}-m}\mathcal{P}_{\mathrm{R}}\epsilon\,. \qquad (44)\end{aligned}$$

For the fourth Green's function we find one diagram



(45)

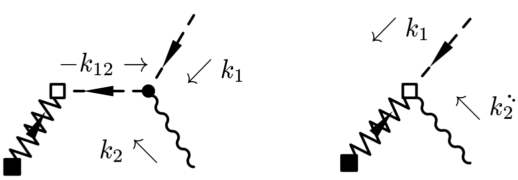
and the expression

$$\begin{aligned}&\frac{\mathrm{i}}{k_1^2-m^2}\frac{\mathrm{i}}{\not{k}_{12}-m}\left(-\mathrm{i}e\sqrt{2}\mathcal{P}_{\mathrm{R}}\right)\frac{\mathrm{i}}{\not{k}_2}\gamma_\nu\epsilon\\ &=-\frac{\sqrt{2}e}{k_1^2-m^2}\frac{1}{\not{k}_{12}-m}\frac{1}{\not{k}_2}\gamma_\nu\mathcal{P}_{\mathrm{R}}\epsilon\,. \qquad (46)\end{aligned}$$

The last Green's function gives two contributions:

$$\begin{aligned}&\sqrt{2}\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)A_\nu(x_2)(\mathrm{i}\not{\partial}+m)\phi_-(x_3)\mathcal{P}_{\mathrm{R}}\epsilon|0\rangle \qquad (47)\\ &+\sqrt{2}e\langle 0|\mathrm{T}\,\phi_-^\dagger(x_1)A_\nu(x_2)\gamma^\lambda(A_\lambda\phi_-)(x_3)\mathcal{P}_{\mathrm{R}}\epsilon|0\rangle\,,\end{aligned}$$

corresponding to the diagrams



(48)

For the right diagram we find

$$\sqrt{2}e\frac{\mathrm{i}}{k_1^2-m^2}\frac{-\mathrm{i}\eta_{\nu\lambda}}{k_2^2}\gamma^\lambda\mathcal{P}_{\mathrm{R}}\epsilon=\frac{\sqrt{2}e}{(k_1^2-m^2)k_2^2}\gamma_\nu\mathcal{P}_{\mathrm{R}}\epsilon\,, \qquad (49)$$

while the left diagram gives the result

$$\begin{aligned}&\sqrt{2}\frac{\mathrm{i}}{k_1^2-m^2}\frac{-\mathrm{i}\eta_{\nu\beta}}{k_2^2}\frac{\mathrm{i}}{k_{12}^2-m^2}\mathrm{i}e(k_1+k_{12})^\beta(\not{k}_{12}+m)\mathcal{P}_{\mathrm{R}}\epsilon\\ &=\frac{-\sqrt{2}e}{(k_1^2-m^2)k_2^2\,(k_{12}^2-m^2)}(2k_1+k_2)_\nu(\not{k}_{12}+m)\mathcal{P}_{\mathrm{R}}\epsilon\,. \qquad (50)\end{aligned}$$

Adding all four terms, we find that the sum vanishes:

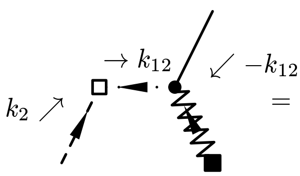
$$\begin{aligned}&\frac{\sqrt{2}e}{(k_1^2-m^2)k_2^2(\not{k}_{12}-m)}\big\{\gamma_\nu(\not{k}_1+m)+k_{2,\nu}-\not{k}_2\gamma_\nu\\ &+(\not{k}_{12}-m)\gamma_\nu-(2k_1+k_2)_\nu\big\}\mathcal{P}_{\mathrm{R}}\epsilon=0\,, \qquad (51)\end{aligned}$$

and this SSTI is also satisfied.

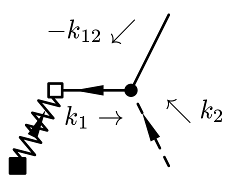
Finally, the SSTI corresponding to the SWI (21) is [8]

$$\begin{aligned}0&=\langle 0|\mathrm{T}\left\{Q_{\mathrm{BRST}},\phi_-(x_1)\phi_-^\dagger(x_2)\lambda(x_3)\right\}|0\rangle\\ &=\mathrm{i}e\langle 0|\mathrm{T}\,c(x_1)\phi_-(x_1)\phi_-^\dagger(x_2)\lambda(x_3)|0\rangle\\ &\quad-\sqrt{2}\langle 0|\mathrm{T}\,(\overline{\epsilon}\mathcal{P}_{\mathrm{L}}\Psi(x_1))\phi_-^\dagger(x_2)\lambda(x_3)|0\rangle\\ &\quad-\mathrm{i}e\langle 0|\mathrm{T}\,\phi_-(x_1)c(x_2)\phi_-^\dagger(x_2)\lambda(x_3)|0\rangle\\ &\quad+\sqrt{2}\langle 0|\mathrm{T}\,\phi_-(x_1)(\overline{\Psi}(x_2)\mathcal{P}_{\mathrm{R}}\epsilon)\lambda(x_3)|0\rangle\\ &\quad+\frac{\mathrm{i}}{2}e\langle 0|\mathrm{T}\,\phi_-(x_1)\phi_-^\dagger(x_2)\partial_\alpha A_\beta(x_3)\left[\gamma^\alpha,\gamma^\beta\right]\epsilon|0\rangle\\ &\quad+e\langle 0|\mathrm{T}\,\phi_-(x_1)\phi_-^\dagger(x_2)\\ &\qquad\times\left(|\phi_-(x_3)|^2-|\phi_+(x_3)|^2\right)\gamma^5\epsilon|0\rangle\,. \qquad (52)\end{aligned}$$

None of these Green's functions vanish and the contributions are

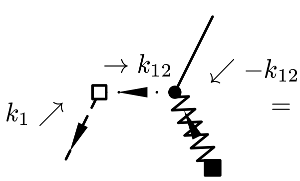


$$=\frac{e}{(k_2^2-m^2)\,k_{12}^2}\epsilon \qquad (53a)$$

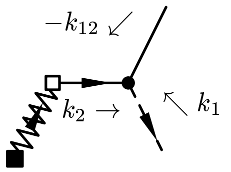


$$=\frac{-2e}{(k_1^2-m^2)\,(k_2^2-m^2)}\frac{1}{\not{k}_{12}}\not{k}_1\mathcal{P}_{\mathrm{L}}\epsilon$$

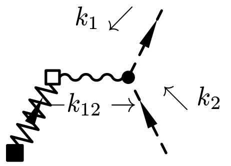
(53b)



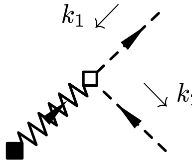
$$=\frac{-e}{(k_1^2-m^2)\,k_{12}^2}\epsilon \qquad (53c)$$



$$= \frac{2e}{(k_1^2 - m^2)\,(k_2^2 - m^2)} \frac{1}{\not k_{12}} \not k_2 \mathcal{P}_{\mathrm{R}} \epsilon \tag{53d}$$



$$= \frac{e[\not k_{12}, \not k_1 - \not k_2]\epsilon}{2\,(k_1^2 - m^2)\,(k_2^2 - m^2)\,k_{12}^2} \tag{53e}$$



$$= \frac{-e}{(k_1^2 - m^2)\,(k_2^2 - m^2)} \gamma^5 \epsilon \tag{53f}$$

After combining these contributions, simple Dirac algebra shows that they add to zero, which proves that this SSTI is valid, too.

These examples have demonstrated how the formalism of SSTIs works for supersymmetric gauge theories, when the constant SUSY ghosts are introduced with the correct couplings. The explicit calculations have demonstrated how all components have to interact in order to satisfy the SSTIs. Testing a set of SSTIs in a model numerically will simultaneously test the Feynman rules, the signs from statistics and the numerics of vertex factors.

4 Non-abelian gauge theories

We can also apply the formalism of BRST quantization to non-abelian supersymmetric gauge theories (see Appendix B.2 for the Lagrangian and our conventions). The gauge part of the BRST transformations contains terms that are absent in the abelian case (27):

$$\begin{aligned} s\phi_{-,i}(x) &= \mathrm{i}gc^a(x)\phi_{-,j}(x)T^a_{ji} \\ &\quad - \sqrt{2}\,(\bar\epsilon \mathcal{P}_{\mathrm{L}} \Psi_i(x)) - \mathrm{i}\omega^\nu \partial_\nu \phi_{-,i}(x)\,, \end{aligned} \tag{54a}$$

$$\begin{aligned} s\phi^\dagger_{-,i}(x) &= -\mathrm{i}gc^a(x)T^a_{ij}\phi^\dagger_{-,j}(x) \\ &\quad + \sqrt{2}\left(\overline{\Psi}_i(x)\mathcal{P}_{\mathrm{R}}\epsilon\right) - \mathrm{i}\omega^\nu \partial_\nu \phi^\dagger_{-,i}(x)\,, \end{aligned} \tag{54b}$$

$$\begin{aligned} s\phi_{+,i}(x) &= -\mathrm{i}gc^a(x)T^a_{ij}\phi_{+,j}(x) \\ &\quad + \sqrt{2}\left(\overline{\Psi}_i(x)\mathcal{P}_{\mathrm{L}}\epsilon\right) - \mathrm{i}\omega^\nu \partial_\nu \phi_{+,i}(x)\,, \end{aligned} \tag{54c}$$

$$\begin{aligned} s\phi^\dagger_{+,i}(x) &= \mathrm{i}gc^a(x)\phi^\dagger_{+,j}(x)T^a_{ji} \\ &\quad - \sqrt{2}\,(\bar\epsilon \mathcal{P}_{\mathrm{R}} \Psi_i(x)) - \mathrm{i}\omega^\nu \partial_\nu \phi^\dagger_{+,i}(x)\,, \end{aligned} \tag{54d}$$

$$\begin{aligned} s\Psi_i(x) &= \mathrm{i}gc^a(x)T^a_{ij}\Psi_j(x) \\ &\quad + \sqrt{2}\Big[(\mathrm{i}\not\partial + m)\phi_{-,i}(x)\mathcal{P}_{\mathrm{R}} - (\mathrm{i}\not\partial + m)\phi^\dagger_{+,i}(x)\mathcal{P}_{\mathrm{L}} \\ &\quad + g\not A^a(x)T^a_{ij}\phi_{-,j}(x)\mathcal{P}_{\mathrm{R}} - g\not A^a(x)T^a_{ij}\phi^\dagger_{+,j}(x)\mathcal{P}_{\mathrm{L}}\Big]\epsilon \\ &\quad - \mathrm{i}\omega^\nu \partial_\nu \Psi_i(x)\,, \end{aligned} \tag{54e}$$

$$\begin{aligned} s\overline{\Psi}_i(x) &= -\mathrm{i}gc^a(x)\overline{\Psi}_j(x)T^a_{ji} \\ &\quad + \sqrt{2}\bar\epsilon\Big[\mathcal{P}_{\mathrm{L}}(\mathrm{i}\not\partial - m)\phi^\dagger_{-,i}(x) - \mathcal{P}_{\mathrm{R}}(\mathrm{i}\not\partial - m)\phi_{+,i}(x) \\ &\quad - g\phi^\dagger_{-,j}(x)\mathcal{P}_{\mathrm{L}}T^a_{ji}\not A^a(x) + g\phi_{+,j}(x)\mathcal{P}_{\mathrm{R}}T^a_{ji}\not A^a(x)\Big] \\ &\quad - \mathrm{i}\omega^\nu \partial_\nu \overline{\Psi}_i(x)\,, \end{aligned} \tag{54f}$$

$$sA^a_\mu(x) = (D_\mu c(x))^a - \bar\epsilon\gamma_\mu\lambda^a(x) - \mathrm{i}\omega^\nu\partial_\nu A^a_\mu(x)\,, \tag{54g}$$

$$\begin{aligned} s\lambda^a(x) &= gf_{abc}c^b(x)\lambda^c(x) + \frac{\mathrm{i}}{2}F^a_{\alpha\beta}(x)\gamma^\alpha\gamma^\beta\epsilon \\ &\quad + g\left(\phi^\dagger_-(x)T^a\phi_-(x)\right)\gamma^5\epsilon \\ &\quad - g\left(\phi_+(x)T^a\phi^\dagger_+(x)\right)\gamma^5\epsilon - \mathrm{i}\omega^\nu\partial_\nu\lambda^a(x)\,, \end{aligned} \tag{54h}$$

$$\begin{aligned} s\overline{\lambda^a}(x) &= gf_{abc}c^b(x)\overline{\lambda^c}(x) - \frac{\mathrm{i}}{2}\bar\epsilon\gamma^\alpha\gamma^\beta F^a_{\alpha\beta}(x) \\ &\quad + g\bar\epsilon\gamma^5\left(\phi^\dagger_-(x)T^a\phi_-(x)\right) \\ &\quad - g\bar\epsilon\gamma^5\left(\phi_+(x)T^a\phi^\dagger_+(x)\right) \\ &\quad - \mathrm{i}\omega^\nu\partial_\nu\overline{\lambda^a}(x)\,, \end{aligned} \tag{54i}$$

$$\begin{aligned} sc^a(x) &= -\frac{g}{2}f_{abc}c^b(x)c^c(x) \\ &\quad + \mathrm{i}(\bar\epsilon\gamma^\mu\epsilon)A^a_\mu(x) - \mathrm{i}\omega^\nu\partial_\nu c^a(x)\,, \end{aligned} \tag{54j}$$

$$s\bar c^a(x) = \mathrm{i}B^a(x) - \mathrm{i}\omega^\nu\partial_\nu\bar c^a(x)\,, \tag{54k}$$

$$sB^a(x) = (\bar\epsilon\gamma^\mu\epsilon)\partial_\mu\bar c^a(x) - \mathrm{i}\omega^\nu\partial_\nu B^a(x)\,, \tag{54l}$$

$$s\epsilon = 0\,, \tag{54m}$$

$$s\omega^\mu = (\bar\epsilon\gamma^\mu\epsilon)\,. \tag{54n}$$

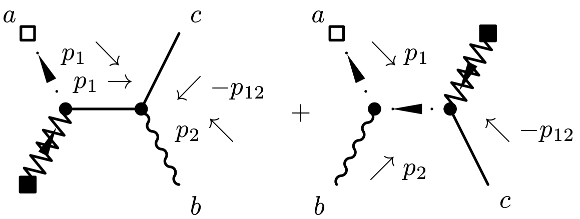
Except for the ghost-ghost–gluon vertex, familiar from non-supersymmetric gauge theories, the gauge-fixing and ghost part of the Lagrangian is the same as in the abelian case (32). In particular, the SUSY ghost interactions are identical to (33), with the obvious sum over the gauge ghost implied.

To demonstrate a SSTI in SYM, we choose two gluons and a gluino, since this will involve the non-abelian coupling of gluons, gluinos and ghosts [10]:

$$\begin{aligned} 0 &\stackrel{!}{=} \langle 0|\mathrm{T}\left\{Q_{\mathrm{BRST}}, A^a_\mu(x_1)A^b_\nu(x_2)\lambda^c(x_3)\right\}|0\rangle \\ &= \langle 0|\mathrm{T}\,(D_\mu c)^a(x_1)A^b_\nu(x_2)\lambda^c(x_3)|0\rangle \\ &\quad - \langle 0|\mathrm{T}\,(\bar\epsilon\gamma_\mu\lambda^a(x_1))\,A^b_\nu(x_2)\lambda^c(x_3)|0\rangle \\ &\quad + (a \leftrightarrow b, \mu \leftrightarrow \nu, x_1 \leftrightarrow x_2) \\ &\quad + \frac{\mathrm{i}}{2}\langle 0|\mathrm{T}\,A^a_\mu(x_1)A^b_\nu(x_2)\partial_\lambda A^c_\kappa(x_3)\left[\gamma^\lambda,\gamma^\kappa\right]\epsilon|0\rangle \\ &\quad + \frac{\mathrm{i}g}{4}\langle 0|\mathrm{T}\,A^a_\mu(x_1)A^b_\nu(x_2)\left(A^e_\lambda A^f_\kappa\right)(x_3)\left[\gamma^\lambda,\gamma^\kappa\right]f^{cef}\epsilon|0\rangle\,. \end{aligned} \tag{55}$$

Two Feynman diagrams contribute to the derivative part of the first Green's function

$$-\,\mathrm{F.T.}\int \mathrm{d}^4z\,\langle 0|\mathrm{T}\,\partial_\mu c^a(x_1)\bar{c}^d(z)\lambda^c(x_3)$$
$$\times\left(\overline{\lambda^d}(z)\,\overleftarrow{\not\partial}\,\epsilon\right)A^b_\nu(x_2)\,|0\rangle \tag{56}$$

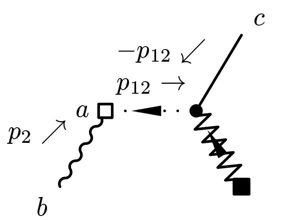
$=$ 

and evaluate to

$$-\frac{-1}{p_1^2}\frac{-\mathrm{i}}{p_2^2}\frac{\mathrm{i}}{\not p_{12}}g\gamma_\nu f^{abc}\frac{\mathrm{i}}{\not p_1}(\mathrm{i}\not p_1)\mathrm{i}p_{1,\mu}\epsilon$$
$$=\frac{-\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}\not p_{12}\gamma_\nu p_{1,\mu}\epsilon\,, \tag{57a}$$
$$-\frac{\mathrm{i}}{\not p_{12}}\mathrm{i}\not p_{12}\epsilon\frac{-1}{p_{12}^2}\cdot(-\mathrm{i}gf^{abc})p_{1,\nu}\frac{-\mathrm{i}}{p_2^2}\frac{-1}{p_1^2}(\mathrm{i}p_{1,\mu})$$
$$=\frac{-\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}p_{1,\mu}p_{1,\nu}\epsilon\,. \tag{57b}$$

The gauge connection contributes another diagram

$$-\,gf^{ade}\mathrm{F.T.}\int \mathrm{d}^4z\,\langle 0|\mathrm{T}\,\left(A^d_\mu c^e\right)(x_1)\bar{c}^f(z)\lambda^c(x_3)$$
$$\times\left(\overline{\lambda^f}(z)\,\overleftarrow{\not\partial}\,\epsilon\right)A^b_\nu(x_2)\,|0\rangle$$

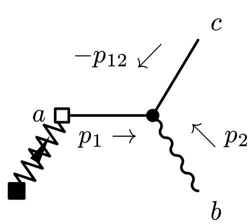
$=$  (58)

which gives the analytical expression

$$-gf^{abc}\frac{-\mathrm{i}}{p_2^2}\eta_{\mu\nu}\frac{-1}{p_{12}^2}\frac{\mathrm{i}}{\not p_{12}}\mathrm{i}\not p_{12}\epsilon=\frac{\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}p_1^2\eta_{\mu\nu}\epsilon\,. \tag{59}$$

The second Green's function results from the SUSY part of the BRST transformation and contributes only a single diagram

$$\mathrm{F.T.}\langle 0|\mathrm{T}\,A^b_\nu(x_2)\lambda^c(x_3)\left(\overline{\lambda^a}(x_1)\gamma_\mu\epsilon\right)|0\rangle$$

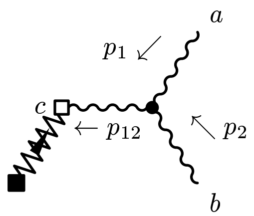
$=$  (60)

This yields the analytical expression

$$\frac{\mathrm{i}}{\not p_{12}}\frac{-\mathrm{i}}{p_{12}^2}g\gamma_\nu f^{abc}\frac{\mathrm{i}}{\not p_1}\gamma_\mu\epsilon=\frac{\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}\not p_{12}\gamma_\nu\not p_1\gamma_\mu\epsilon\,. \tag{61}$$

The final two Green's functions are contributed by the SUSY transformation of the gluino, the first

$$\frac{\mathrm{i}}{2}\mathrm{F.T.}\langle 0|\mathrm{T}\,A^a_\mu(x_1)A^b_\nu(x_2)\partial_\lambda A^c_\kappa(x_3)\left[\gamma^\lambda,\gamma^\kappa\right]\epsilon|0\rangle$$

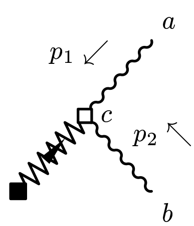
$=$  (62)

yielding

$$\frac{\mathrm{i}}{2}\frac{-\mathrm{i}}{p_1^2}\frac{-\mathrm{i}}{p_2^2}\frac{-\mathrm{i}}{p_{12}^2}gf^{abc}(-\mathrm{i})p_{12,\lambda}\left[\gamma^\lambda,\gamma^\kappa\right]$$
$$\times\left[\eta_{\mu\nu}\left(p_1-p_2\right)_\kappa+\eta_{\nu\kappa}\left(2p_2+p_1\right)_\mu+\eta_{\mu\kappa}\left(-2p_1-p_2\right)_\nu\right]\epsilon$$
$$=\frac{\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}\frac{1}{2}\Big[\eta_{\mu\nu}[\not p_{12},\not p_1-\not p_2]+(2p_2+p_1)_\mu[\not p_{12},\gamma_\nu]$$
$$-(2p_1+p_2)_\nu[\not p_{12},\gamma_\mu]\Big]\epsilon\,, \tag{63}$$

and the second one

$$\frac{\mathrm{i}gf^{cde}}{4}\mathrm{F.T.}\langle 0|\mathrm{T}\,A^a_\mu(x_1)A^b_\nu(x_2)\left(A^d_\lambda A^e_\kappa\right)(x_3)\left[\gamma^\lambda,\gamma^\kappa\right]\epsilon\,|0\rangle$$

$=$  (64)

yielding (with a symmetry factor 2!)

$$\frac{\mathrm{i}}{2}gf^{abc}\frac{-\mathrm{i}}{p_1^2}\frac{-\mathrm{i}}{p_2^2}[\gamma_\mu,\gamma_\nu]\epsilon=\frac{-\mathrm{i}gf^{abc}}{p_1^2p_2^2p_{12}^2}\frac{1}{2}[\gamma_\mu,\gamma_\nu]p_{12}^2\epsilon\,. \tag{65}$$

Collecting all the contributions (including the symmetrization), we find that they indeed add to zero and the SSTI is satisfied, as expected. This example shows the non-trivial cancellations among the gauge and the SUSY parts of the BRST transformations, which are at work already for very simple Feynman diagrams.

5 Conclusions

In this paper, we have revisited the off-shell non-conservation of the supersymmetric current in supersymmetric gauge theories. The BRST formalism allows one to derive supersymmetric Slavnov–Taylor identities, which can replace the supersymmetric Ward identities. The SWIs are violated off-shell as a result of perturbative gauge fixing, while the SSTIs remain valid with the help of additional ghost interactions.

The investigation of the diagrammatical structure of the SSTIs shows that they provide efficient consistency checks for the implementation of supersymmetric gauge theories in matrix element generators [17]. It is possible to generate all SSTIs for a given number of external particles systematically and test them numerically. This procedure detects flaws in the implementation of Feynman rules and in the numerical stability with great sensitivity [12].

Since the identities depend on the conservation of the BRST charge and not on properties of the ground state, the formalism can also be applied to spontaneously broken symmetries. For the phenomenologically important case of softly broken SUSY [18], the explicit breaking has to be implemented using a spurion formalism, the practical application of which requires further studies. Our diagrammatical results can also be used as a basis for constructing supersymmetric subsets of Feynman diagrams along the lines of [5, 19].

Acknowledgements. We thank C. Schwinn and K. Sibold for helpful discussions. This research is supported by Bundesministerium für Bildung und Forschung, Germany, (05 HT9RDA).

A Notation and conventions

A.1 Majorana spinors

For phenomenological applications with massive particles, four component spinors are more convenient. Our Majorana spinors satisfy

$$\Psi^c \equiv \mathcal{C}\overline{\Psi}^{\mathrm{T}} = \Psi\,, \tag{66}$$

with $\mathcal{C} = \mathrm{i}\gamma^2\gamma^0$ the antisymmetric charge conjugation matrix. In the sequel, θ will always denote a Grassmann odd spinor. Then we have

$$\begin{aligned}\overline{\theta}_1\Gamma\theta_2 &= \left(\overline{\theta}_1\Gamma\theta_2\right)^{\mathrm{T}} = -\left(\theta_1^{\mathrm{T}}\mathcal{C}\Gamma\theta_2\right)^{\mathrm{T}} \\ &= -\left(\theta_2^{\mathrm{T}}\Gamma^{\mathrm{T}}\mathcal{C}\theta_1\right) = \overline{\theta}_2\mathcal{C}^{-1}\Gamma^{\mathrm{T}}\mathcal{C}\theta_1\,.\end{aligned} \tag{67}$$

Using

$$\Gamma^{\mathrm{T}} = \begin{cases} +\mathcal{C}\Gamma\mathcal{C}^{-1}\,, & \Gamma = \mathbb{I}, \gamma^5\gamma^\mu, \gamma^5\,, \\ -\mathcal{C}\Gamma\mathcal{C}^{-1}\,, & \Gamma = \gamma^\mu, [\gamma^\mu, \gamma^\nu]\,, \end{cases} \tag{68}$$

we have

$$\overline{\theta}_1\Gamma\theta_2 = \begin{cases} +\overline{\theta}_2\Gamma\theta_1\,, & \Gamma = \mathbb{I}, \gamma^5\gamma^\mu, \gamma^5\,, \\ -\overline{\theta}_2\Gamma\theta_1\,, & \Gamma = \gamma^\mu, [\gamma^\mu, \gamma^\nu]\,, \end{cases} \tag{69}$$

but for commuting spinors like the SUSY ghosts, the signs in (69) are reversed.

A.2 SUSY transformations

The SUSY transformations for chiral and vector superfields read

$$\delta_\xi\phi = \sqrt{2}\left(\overline{\xi_{\mathrm{R}}}\psi_{\mathrm{L}}\right)\,, \tag{70a}$$

$$\delta_\xi\psi_{\mathrm{L}} = -\sqrt{2}\mathrm{i}(\not{\partial}\phi)\xi_{\mathrm{R}} - \sqrt{2}\left(\frac{\partial\mathcal{W}(\phi)}{\partial\phi}\right)^*\xi_{\mathrm{L}} \tag{70b}$$

and

$$\delta_\xi A^a_\mu = -\left(\overline{\xi}\gamma_\mu\gamma^5\lambda^a\right)\,, \tag{71a}$$

$$\delta_\xi\lambda^a = -\frac{\mathrm{i}}{4}\left[\gamma^\alpha, \gamma^\beta\right]\gamma^5 F^a_{\alpha\beta}\xi - e\left(\phi^\dagger T^a\phi\right)\xi\,, \tag{71b}$$

where $\mathcal{W}$ is the superpotential.

B Models

B.1 Supersymmetric quantum electrodynamics (SQED)

In our conventions $\hat{\Phi}_-$ is a left-handed superfield with charge $-e$, while $\hat{\Phi}_+$ is a right-handed superfield with the opposite charge. The covariant derivative is

$$D_\mu = \partial_\mu - \mathrm{i}eA_\mu \tag{72}$$

with e being the modulus of the electron's charge.

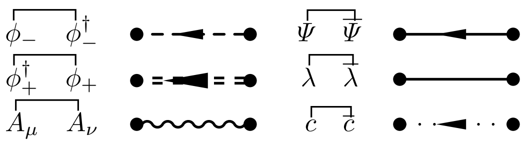
We diagonalize the mass terms of the fermions by introducing the bispinors as the usual electron

$$\Psi = \begin{pmatrix}\psi_- \\ \bar{\psi}_+\end{pmatrix}, \qquad \overline{\Psi} = \left(\psi_+, \bar{\psi}_-\right)\,. \tag{73}$$

By the redefinitions of the fermion fields and after integrating out all auxiliary fields we get the Lagrangian density (including gauge-fixing, Faddeev–Popov terms and SUSY ghosts)

$$\begin{aligned}\mathcal{L} = {} & (D_\mu\phi_+)^\dagger(D^\mu\phi_+) - m^2|\phi_+|^2 \\ & + (D_\mu\phi_-)^\dagger(D^\mu\phi_-) - m^2|\phi_-|^2 + \overline{\Psi}(\mathrm{i}\not{D} - m)\Psi \\ & + \frac{\mathrm{i}}{2}\overline{\lambda}(\not{\partial}\lambda) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \sqrt{2}e(\overline{\Psi}\mathcal{P}_{\mathrm{L}}\lambda)\phi_+^\dagger - \sqrt{2}e(\overline{\Psi}\mathcal{P}_{\mathrm{R}}\lambda)\phi_- \\ & + \sqrt{2}e(\overline{\lambda}\mathcal{P}_{\mathrm{R}}\Psi)\phi_+ - \sqrt{2}e(\overline{\lambda}\mathcal{P}_{\mathrm{L}}\Psi)\phi_-^\dagger \\ & - \frac{e^2}{2}\left(|\phi_+|^2 - |\phi_-|^2\right)^2 - \frac{1}{2\xi}(\partial^\mu A_\mu)(\partial^\nu A_\nu) \\ & + \mathrm{i}\overline{c}\Box c - \mathrm{i}\overline{c}(\overline{\epsilon}\not{\partial}\lambda) + \frac{\mathrm{i}\xi}{2}\overline{c}(\overline{\epsilon}\gamma^\mu\epsilon)\partial_\mu\overline{c}\,.\end{aligned} \tag{74}$$

Our conventions for the particle propagators in the Feynman rules are (the arrows indicate the flow of the charge $-e$ or of ghost number):



(75)

With all momenta incoming, the vertices are

$$
\begin{array}{rcl}
A_\mu \phi_-(p_1)\phi_-^\dagger(p_2) & : & \mathrm{i}e\,(p_1-p_2)_\mu \\
A_\mu \phi_+^\dagger(p_1)\phi_+(p_2) & : & \mathrm{i}e\,(p_1-p_2)_\mu \\
A_\mu \overline{\Psi}\Psi & : & \mathrm{i}e\gamma_\mu \\
\phi_-\overline{\Psi}\lambda & : & -\sqrt{2}\mathrm{i}e\mathcal{P}_\mathrm{R} \\
\phi_-^\dagger\overline{\lambda}\Psi & : & -\sqrt{2}\mathrm{i}e\mathcal{P}_\mathrm{L} \\
\phi_+\overline{\lambda}\Psi & : & \sqrt{2}\mathrm{i}e\mathcal{P}_\mathrm{R} \\
\phi_+^\dagger\overline{\Psi}\lambda & : & \sqrt{2}\mathrm{i}e\mathcal{P}_\mathrm{L} \\
\overline{c}(-p)\overline{\epsilon}\lambda(p) & : & -\mathrm{i}\not{p} \\
|\phi_-|^2 A_\mu A_\nu & : & 2\mathrm{i}e^2\eta_{\mu\nu} \\
|\phi_+|^2 A_\mu A_\nu & : & 2\mathrm{i}e^2\eta_{\mu\nu} \\
\left(|\phi_-|^2\right)^2 & : & -2\mathrm{i}e^2 \\
\left(|\phi_+|^2\right)^2 & : & -2\mathrm{i}e^2 \\
|\phi_-|^2|\phi_+|^2 & : & \mathrm{i}e^2 \\
\epsilon\overline{c}(p)\overline{c}(-p)\overline{\epsilon} & : & \xi\not{p}
\end{array}
\tag{76}
$$

B.2 Supersymmetric Yang–Mills theory (SYM)

Generalizing the abelian case, we introduce a superfield $\hat{\Phi}_-$ transforming under a representation T^a of some non-abelian gauge group and a superfield $\hat{\Phi}_+$ transforming under the conjugate representation $-(T^a)^*$. The generators of the gauge group fulfill the Lie algebra

$$
\left[T^a,T^b\right]=\mathrm{i}f_{abc}T^c,\quad \left[(-T^a)^*,(-T^b)^*\right]=\mathrm{i}f_{abc}(-T^c)^*\,. \tag{77}
$$

As for SQED, we diagonalize the mass terms of the fermions by introducing the bispinors

$$
\Psi_i=\begin{pmatrix}\psi_{-,i}\\ \bar{\psi}_{+,i}\end{pmatrix},\qquad \overline{\Psi}_i=\left(\psi_{+,i},\bar{\psi}_{-,i}\right)\,. \tag{78}
$$

By the redefinitions of the fermion fields and after integrating out all auxiliary fields we get the Lagrangian density (including gauge-fixing, Faddeev–Popov terms and SUSY ghosts):

$$
\begin{aligned}
\mathcal{L}=&\,(D_\mu\phi_+)^\dagger(D^\mu\phi_+)-m^2|\phi_+|^2\\
&+(D_\mu\phi_-)^\dagger(D^\mu\phi_-)-m^2|\phi_-|^2+\overline{\Psi}(\mathrm{i}\not{D}-m)\Psi\\
&+\frac{\mathrm{i}}{2}\overline{\lambda^a}(\not{D}\lambda)^a-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}\\
&-\sqrt{2}g\phi^\dagger_{-,i}T^a_{ij}(\overline{\lambda^a}\mathcal{P}_\mathrm{L}\Psi_j)+\sqrt{2}g\phi_{+,i}T^a_{ij}(\overline{\lambda^a}\mathcal{P}_\mathrm{R}\Psi_j)\\
&-\sqrt{2}g(\overline{\Psi}_i\mathcal{P}_\mathrm{R}\lambda^a)T^a_{ij}\phi_{-,j}+\sqrt{2}g(\overline{\Psi}_i\mathcal{P}_\mathrm{L}\lambda^a)T^a_{ij}\phi^\dagger_{+,j}\\
&-\frac{g^2}{2}\left(\phi^\dagger_{-,i}T^a_{ij}\phi_{-,j}\right)\left(\phi^\dagger_{-,k}T^a_{kl}\phi_{-,l}\right)\\
&-\frac{g^2}{2}\left(\phi_{+,i}T^a_{ij}\phi^\dagger_{+,j}\right)\left(\phi_{+,k}T^a_{kl}\phi^\dagger_{+,l}\right)\\
&+g^2\left(\phi^\dagger_{-,i}T^a_{ij}\phi_{-,j}\right)\left(\phi_{+,k}T^a_{kl}\phi^\dagger_{+,l}\right)\\
&-\frac{1}{2\xi}(\partial^\mu A^a_\mu)(\partial^\nu A^a_\nu)+\mathrm{i}\overline{c}^a\partial_\mu(D^\mu c)^a\\
&-\mathrm{i}\overline{c}^a(\overline{\epsilon}\not{\partial}\lambda^a)+\frac{\mathrm{i}\xi}{2}\overline{c}^a(\overline{\epsilon}\gamma^\mu\epsilon)\partial_\mu\overline{c}^a\,.
\end{aligned}
\tag{79}
$$

The propagators are identical to (75), while the three-point vertices are (with all momenta incoming):

$$
\begin{array}{rcl}
 & & gf_{abc}[\quad\eta_{\mu\nu}(p_1-p_2)_\rho \\
A^a_\mu(p_1)A^b_\nu(p_2)A^c_\rho(p_3) & : & \quad+\eta_{\nu\rho}(p_2-p_3)_\mu \\
 & & \quad+\eta_{\rho\mu}(p_3-p_1)_\nu] \\
A^a_\mu\phi_{-,j}(p_1)\phi^\dagger_{-,i}(p_2) & : & \mathrm{i}g\,(p_1-p_2)_\mu\,T^a_{ij} \\
A^a_\mu\phi^\dagger_{+,j}(p_1)\phi_{+,i}(p_2) & : & \mathrm{i}g\,(p_3-p_2)_\mu\,T^a_{ij} \\
A^a_\mu\overline{\Psi}_i\Psi_j & : & \mathrm{i}g\gamma_\mu T^a_{ij} \\
A^a_\mu\overline{\lambda^b}\lambda^c & : & g\gamma_\mu f_{abc} \\
\phi^\dagger_{-,i}\overline{\lambda^a}\Psi_j & : & -\sqrt{2}\mathrm{i}gT^a_{ij}\mathcal{P}_\mathrm{L} \\
\phi_{+,i}\overline{\lambda^a}\Psi_j & : & \sqrt{2}\mathrm{i}gT^a_{ij}\mathcal{P}_\mathrm{R} \\
\phi_{-,j}\overline{\Psi}_i\lambda^a & : & -\sqrt{2}\mathrm{i}gT^a_{ij}\mathcal{P}_\mathrm{R} \\
\phi^\dagger_{+,j}\overline{\Psi}_i\lambda^a & : & \sqrt{2}\mathrm{i}gT^a_{ij}\mathcal{P}_\mathrm{L} \\
A^b_\mu c^c\overline{c}^a(p) & : & -\mathrm{i}gf_{abc}p_\mu \\
\overline{c}^a(-p)\lambda^b(p)\overline{\epsilon} & : & -\mathrm{i}\not{p}\delta_{ab}
\end{array}
\tag{80}
$$

and we have in addition the following four-point vertices

$$
\begin{array}{rcl}
 & & -\mathrm{i}g^2[\quad f_{abe}f_{cde}(\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}) \\
A^a_\mu A^b_\nu A^c_\rho A^d_\sigma & : & \quad+f_{ace}f_{bde}(\eta_{\mu\nu}\eta_{\rho\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}) \\
 & & \quad+f_{ade}f_{bce}(\eta_{\mu\nu}\eta_{\rho\sigma}-\eta_{\mu\rho}\eta_{\nu\sigma})] \\
\phi_{-,i}\phi^\dagger_{-,j}A^a_\mu A^b_\nu & : & \mathrm{i}g^2\eta_{\mu\nu}\left\{T^a,T^b\right\}_{ij} \\
\phi^\dagger_{+,i}\phi_{+,j}A^a_\mu A^b_\nu & : & \mathrm{i}g^2\eta_{\mu\nu}\left\{T^a,T^b\right\}_{ij} \\
\phi_{-,j}\phi^\dagger_{-,i}\phi_{-,l}\phi^\dagger_{-,k} & : & -\frac{\mathrm{i}g^2}{4}\left(\delta_{il}\delta_{jk}-\frac{1}{N}\delta_{ij}\delta_{kl}\right) \\
\phi^\dagger_{+,j}\phi_{+,i}\phi^\dagger_{+,l}\phi_{+,k} & : & -\frac{\mathrm{i}g^2}{4}\left(\delta_{il}\delta_{jk}-\frac{1}{N}\delta_{ij}\delta_{kl}\right) \\
\phi_{-,j}\phi^\dagger_{-,i}\phi^\dagger_{+,l}\phi_{+,k} & : & \frac{\mathrm{i}g^2}{2}\left(\delta_{il}\delta_{jk}-\frac{1}{N}\delta_{ij}\delta_{kl}\right) \\
\epsilon\overline{c}^a(-p)\overline{\epsilon}\overline{c}^b(p) & : & \xi\not{p}\delta_{ab}
\end{array}
\tag{81}
$$

C Ghosts

Since the SUSY and translation ghosts are not yet as familiar as the Faddeev–Popov ghosts and the relations studied in the main text depend sensitively on the correct choice of signs, we discuss here our conventions in detail.

Starting from a gauge transformation with real parameter $\theta^{a*}=\theta^a$, the ghost of a gauge symmetry can be derived by splitting a Grassmann odd, constant parameter λ off the gauge parameter. This results in a Grassmann odd field, the Faddeev–Popov ghost. Since ghost and anti-ghost can not be hermitian adjoints of each other, we choose both as independent, real fields. Then the parameter λ must be chosen purely imaginary:

$$\mathbb{R} \ni \theta^a = \lambda c^a : \quad (\lambda c^a)^* = c^a \lambda^* = -\lambda^* c^a \;\Rightarrow\; \lambda^* = -\lambda \,. \tag{82}$$

Proceeding analogously for the SUSY transformation parameters ξ^α, $\bar{\xi}_{\dot\alpha}$ (starting in two component notation), we can set our conventions for the SUSY ghosts ϵ^α, $\bar{\epsilon}_{\dot\alpha}$. From

$$\xi^\alpha = \lambda \epsilon^\alpha , \tag{83}$$

and the reality conditions $(\xi^\alpha)^* = \bar{\xi}^{\dot\alpha}$ and $(\epsilon^\alpha)^* = \bar{\epsilon}^{\dot\alpha}$, we get

$$(\xi^\alpha)^* = (\lambda \epsilon^\alpha)^* = \lambda^* (\epsilon^\alpha)^* = -\lambda \bar{\epsilon}^{\dot\alpha} \stackrel{!}{=} \bar{\xi}^{\dot\alpha} , \tag{84}$$

i.e.

$$\xi^\alpha = \lambda \epsilon^\alpha , \qquad \bar{\xi}_{\dot\alpha} = -\lambda \bar{\epsilon}_{\dot\alpha} . \tag{85}$$

Switching to bispinor notation

$$\xi \equiv \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot\alpha} \end{pmatrix} , \qquad \epsilon \equiv \begin{pmatrix} \epsilon_\alpha \\ \bar{\epsilon}^{\dot\alpha} \end{pmatrix} , \tag{86}$$

we arrive finally at

$$\xi = -\lambda \gamma^5 \epsilon \,. \tag{87}$$

The analogous relation for the translation ghosts is derived from infinitesimal translations

$$\delta_a f(x) = a^\mu \partial_\mu f(x) \,, \tag{88}$$

and following the conventions of [8, 9]

$$a^\mu = \mathrm{i} \lambda \omega^\mu \tag{89}$$

for the connection between transformation parameter and translation ghost. The translation is a bosonic symmetry and the translation ghost ω^μ is a Grassmann odd vector. From the reality of the transformation parameter a^μ we can now conclude with

$$\begin{aligned} \mathbb{R}^4 \ni a^\mu \;\Rightarrow\; (\mathrm{i}\lambda\omega^\mu)^* &= -\mathrm{i}\omega^{\mu *} \lambda^* \\ &= +\mathrm{i}\lambda^* \omega^{\mu *} = -\mathrm{i}\lambda\omega^{\mu *} \stackrel{!}{=} \mathrm{i}\lambda\omega^\mu \end{aligned} \tag{90}$$

that

$$\omega^{\mu *} = -\omega^\mu \,. \tag{91}$$

References

1. E. Accomando et al. [ECFA/DESY LC Physics Working Group Collaboration], Phys. Rept. **299**, 1 (1998) [hep-ph/9705442]
2. J.A. Aguilar-Saavedra et al., [ECFA/DESY LC Physics Working Group Collaboration], TESLA Technical Design Report Part III: Physics at an e^+e^- Linear Collider [hep-ph/0106315]
3. T. Abe et al., Resource Book for Snowmass 2001, 30 June–21 July 2001, Snowmass, Colorado, SLAC-R-570
4. T. Ohl, Nucl. Instrum. Meth. A **502**, 818 (2003) [hep-ph/0211058]
5. T. Ohl, C. Schwinn, WUE-ITP-2003-004, IKDA-2003-05 [hep-ph/0305334]
6. O. Piguet, K. Sibold, Renormalized supersymmetry (Birkhäuser, Boston 1986)
7. N. Maggiore, O. Piguet, S. Wolf, Nucl. Phys. B **458**, 403 (1996) [Erratum B **469**, 513 (1996)] [hep-th/9507045]; Nucl. Phys. B **476**, 329 (1996) [hep-th/9604002]
8. W. Hollik, E. Kraus, D. Stöckinger, Eur. Phys. J. C **11**, 365 (1999) [hep-ph/9907393]
9. C. Rupp, R. Scharf, K. Sibold, Nucl. Phys. B **612**, 313 (2001) [hep-th/0101153]
10. W. Hollik, D. Stöckinger, Eur. Phys. J. C **20**, 105 (2001) [hep-ph/0103009]
11. T. Ohl, O'Mega: An Optimizing Matrix Element Generator, in Proceedings of 7th International Workshop on Advanced Computing and Analysis Techniques in Physics Research (ACAT 2000) (Fermilab, Batavia, Il, 2000) [hep-ph/0011243]; M. Moretti, T. Ohl, J. Reuter, hep-ph/0102195
12. J. Reuter, Supersymmetry of Scattering Amplitudes and Green Functions in Perturbation Theory, Ph.D. thesis (TU Darmstadt, 2002) [hep-th/0212154]
13. D.M. Capper, D.R. Jones, P. van Nieuwenhuizen, Nucl. Phys. B **167**, 479 (1980)
14. P.L. White, Class. Quant. Grav. **9**, 1663 (1992)
15. B. de Wit, D.Z. Freedman, Phys. Rev. D **12**, 2286 (1975)
16. U. Theis, P. Van Nieuwenhuizen, Class. Quant. Grav. **18**, 5469 (2001) [hep-th/0108204]
17. T. Ohl, J. Reuter, C. Schwinn, O'Mega (long write-up, in preparation)
18. W. Hollik, E. Kraus, M. Roth, C. Rupp, K. Sibold, D. Stöckinger, Nucl. Phys. B **639**, 3 (2002) [hep-ph/0204350]
19. E. Boos, T. Ohl, Phys. Rev. Lett. **83**, 480 (1999) [hep-ph/9903357]